PHYS 320 ANALYTICAL MECHANICS

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Fall 2108

TODAY

Newton's Laws

Solving the second law: Separation of variables

Review of statics

Statics: next steps … trusses

NEWTON'S LAWS

Valid only in INERTIAL reference frames!

I Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impresses upon it.

II The change of motion is proportional to the motive force impresses; and is made in the direction of the line in which that force is impressed.

Note: these two refer to a specific body (mass)

Newton's Second Law

$$\vec{F}_{net} = \sum_i \vec{F}_i = \frac{d\vec{p}}{dt}$$

include only *physical forces:*

$\vec{F}_{\text{on}\_\_\_\_\_\text{by}\_\_\_\_\_}$

If *m* = constant, then

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2}$$

Newton's Second Law

If m = constant, then for one dimensional motion

$$F_{net} = \sum_i F_i = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}$$

If a = constant, then we can perform **separation of variables** (*ref*. P1.24 & P1.25)

$$ma = m\frac{dv}{dt}$$

$$a\,dt = dv \quad \Rightarrow \quad \int_{t_i}^{t_f} a\,dt = \int_{v_i}^{v_f} dv$$

$$\Rightarrow \quad a(t_f - t_i) = v_f - v_i \qquad \boxed{\therefore\ v_f = v_i + a\,\Delta t}$$

Newton's Second Law

Or, $\boxed{v = v_o + a\,t}$ $(with\ t_i = 0, v_i = v_o, v_f = v)$

Still with a = constant, perform another **separation of variables:**

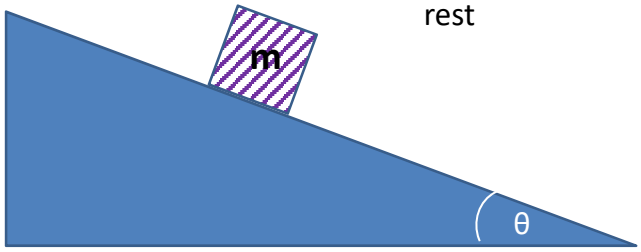
$$v = \frac{dx}{dt} = v_o + a\,t$$

$$dx = (v_o + a\,t)dt \quad \Rightarrow \quad \int_{x_i}^{x_f} dx = \int_{t_i=0}^{t_f=t} (v_o + a\,t)dt$$

$$x_f - x_i = v_o t + \frac{1}{2}a\,t^2 \quad \Rightarrow \quad \boxed{x = x_o + v_o t + \frac{1}{2}a\,t^2}$$

EX: Motion on an inclined plane

- Assume there is friction between box and ramp
- Pushed vs. released from rest



Newton's Second Law

- Suppose $F_{net} = ma \neq constant.$
- Suppose $F_{net} = F(t)$:

$$F(t) = m\,a = m\,\dot{x} = m\,\ddot{x}$$

then

$$\dot{x}(t) = \frac{1}{m}\int_{t_i}^{t_f} F(t)\,dt$$

separation of variables!

Can separate variables and integrate one more time to find $x(t)$!!

Newton's Second Law

Consider HW04, AQII(a): $F(t) = F_o + ct$

Find *v(t)* and *x(t)*.

PAIR UP!!

HW04, AQII(b): $F(t) = F_o \sin(ct)$

HW04, AQII(c): $F(t) = F_o e^{ct}$